

Measuring the Zeeman splitting of the red cadmium line as a function of the magnetic field – spectroscopy using a Fabry-Perot etalon

Objects of the experiment

- Observing the Zeeman splitting with a Fabry-Perot interferometer – using VideoCom
- Determining Bohr's magneton μ_B and the specific electron charge $\frac{e}{m_e}$

Principles

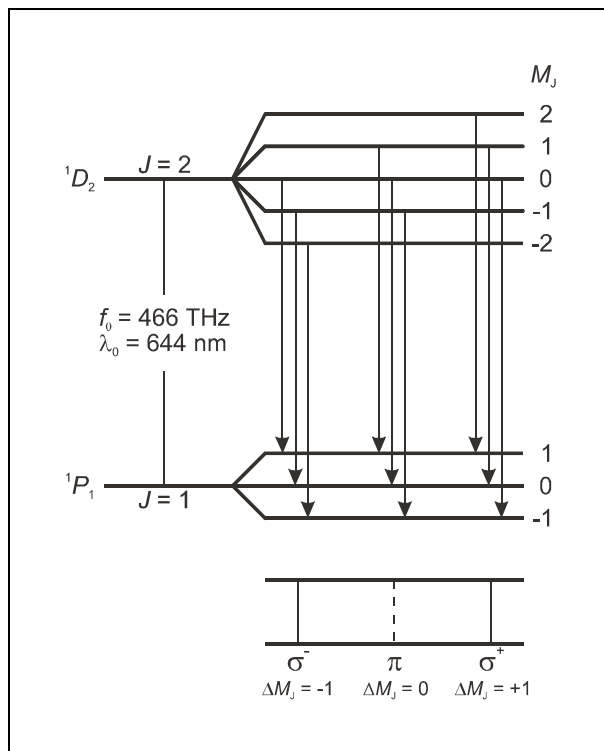


Fig. 1: Level splitting and transitions of the normal Zeeman effect in cadmium

The Zeeman effect is the name for the splitting of atomic energy levels or spectral lines due to the action of an external magnetic field. The effect was first predicted by H. A. Lorentz in 1895 as part of his classic theory of the electron, and experimentally confirmed some years later by P. Zeeman. Zeeman observed a line triplet instead of a single spectral line at right angles to a magnetic field, and a line doublet parallel to the magnetic field. Later, more complex splittings of spectral lines were observed, which became known as the anomalous Zeeman effect. To explain this phenomenon, Goudsmit and

Uhlenbeck first introduced the hypothesis of electron spin in 1925. Ultimately, it became apparent that the anomalous Zeeman effect was actually the rule and the “normal” Zeeman effect the exception.

The normal Zeeman effect only occurs at the transitions between atomic states with the total spin $S = 0$. The total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ of a state is then a pure orbital angular momentum ($\mathbf{J} = \mathbf{L}$). For the corresponding magnetic moment, we can simply say that:

$$\mu = \frac{\mu_B}{\hbar} \mathbf{J} \quad (\text{I})$$

where

$$\mu_B = \frac{\hbar e}{2m_e} \quad (\text{II})$$

($\mu_B =$ Bohr's magneton, $m_e =$ mass of electron, $e =$ elementary charge, $\hbar = h/2\pi$, $h =$ Planck's constant).

In an external magnetic field \mathbf{B} , the magnetic moment has the energy

$$E = -\mu \cdot \mathbf{B} \quad (\text{III})$$

The angular-momentum component in the direction of the magnetic field can have the values

$$J_z = M_J \cdot \hbar \quad \text{with } M_J = J, J-1, \dots, -J \quad (\text{IV})$$

Therefore, the term with the angular momentum J is split into $2J + 1$ equidistant Zeeman components which differ by the value of M_J . The energy interval of the adjacent components M_J, M_{J+1} is

$$\Delta E = -\mu_B \cdot B \quad (\text{V}).$$

We can observe the normal Zeeman effect e.g. in the red spectral line of cadmium ($\lambda_0 = 643,8 \text{ nm}$, $f_0 = 465,7 \text{ THz}$). It corresponds to the transition $^1D_2 (J = 2, S = 0) \rightarrow ^1P_1 (J = 1, S = 0)$ of an electron of the fifth shell (see Fig. 1). In the magnetic field, the 1D_2 level splits into five Zeeman components,

and the level 1P_1 splits into three *Zeeman* components having the spacing calculated using equation (V).

Optical transitions between these levels are only possible in the form of electrical dipole radiation. The following selection rules apply for the magnetic quantum numbers M_J of the states involved:

$$\begin{aligned} \Delta M_J &= \pm 1 && \text{for } \sigma \text{ components} \\ &= 0 && \text{for } \pi \text{ components} \end{aligned} \quad (\text{VI})$$

Thus, we observe a total of three spectral lines (see Fig. 1); the π component is not shifted and the two σ components are shifted by

$$\Delta f = \pm \frac{\Delta E}{h} \quad (\text{VII})$$

with respect to the original frequency. In this equation, ΔE is the equidistant energy splitting calculated in (V).

The *Zeeman* effect enables spectroscopic separation of the differently polarized components. To demonstrate the shift, however, we require a spectral apparatus with extremely high resolution, as the two σ components of the red cadmium line are shifted e.g. at a magnetic flux density $B = 1$ T by only $\Delta f = 14$ GHz, respectively $\Delta\lambda = 0,02$ nm.

In the experiment a *Fabry-Perot* etalon is used. This is a glass plate which is coplanar to a very high precision with both sides being aluminized. The slightly divergent light enters the etalon, which is aligned perpendicularly to the optical axis, and is reflected back and forth several times, whereby part of it emerges each time (see Fig. 2). Due to the aluminizing this emerging part is small, i.e., many emerging rays can interfere. Behind the etalon the emerging rays are focused by a lens on to the focal plane of the lens. There a concentric circular fringe pattern associated with a particular wavelength λ can be observed with an ocular. The aperture angle of a ring is identical with the angle of emergence α of the partial rays from the *Fabry-Perot* etalon.

The rays emerging at an angle of α_k interfere constructively with each other when two adjacent rays fulfil the condition for "curves of equal inclination" (see Fig. 4):

$$\Delta = 2d \cdot \sqrt{n^2 - \sin^2 \alpha_k} = k \cdot \lambda \quad (\text{VIII})$$

(Δ = optical path difference, d = thickness of the etalon, n = refractive index of the glass, k = order of interference)).

A change in the wavelength by $\delta\lambda$ is seen as a change in the aperture angle by $\delta\alpha$. Depending on the focal length of the lens, the aperture angle α corresponds to a radius r and the change in the angle $\delta\alpha$ to a change in the radius δr . If a spectral line contains several components with the distance $\delta\lambda$, each circular interference fringe is split into as many components with the radial distance δr . So a spectral line doublet is recognized by a doublet structure and a spectral line triplet by a triplet structure in the circular fringe pattern.

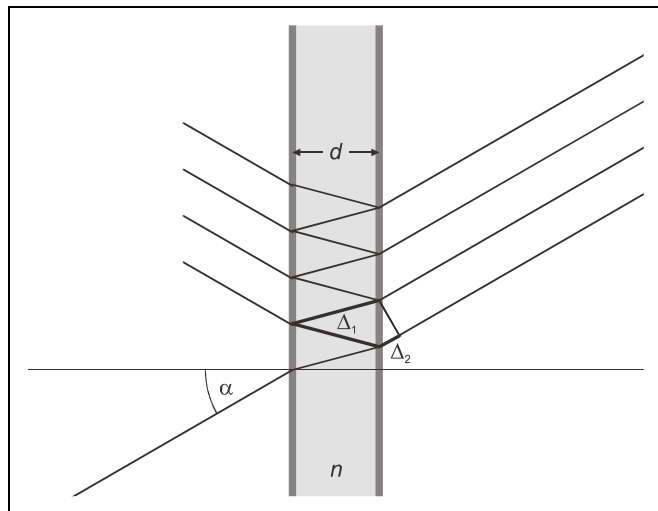


Fig. 2: *Fabry-Perot* etalon as an interference spectrometer.

The ray path is drawn for an angle $\alpha > 0$ relative to the optical axis. The optical path difference between two adjacent emerging rays is $\Delta = n \cdot \Delta_1 - \Delta_2$.

In the experiment, the *Zeeman* splitting of the red cadmium line is measured as a function of the magnetic field B . The energy interval of the triplet components (see equations (II) and (V))

$$E = \frac{h}{4\pi} \frac{e}{m_e} \cdot B \quad (\text{VIII})$$

is used to calculate the specific electron charge.

To achieve spectroscopic separation of the *Zeeman* components of the red cadmium spectral line, the optical ring system of a *Fabry-Pérot* interferometer is imaged on the Video-Com CCD (without camera lens) using an imaging lens. The CCD detects a diametrical section through the optical ring system.

Apparatus

1 Cadmium lamp	451 12
1 Universal choke in housing	451 30
1 High current power supply	521 55
1 U-core with yoke	562 11
2 Coils, 10 A, 480 turns	562 131
1 Pair of pole pieces with great bore	560 315
1 Fabry-Perot etalon	471 221
1 Precision optical bench, 1 m	460 32
1 Rider base with threads	460 381
5 Optics riders 60/50	460 373
2 Lenses, $f = +150$ mm	460 08
1 Holder for interference filters	468 41
1 Interference filter, 644 nm	468 400
1 VideoCom USB	337 47USB
1 Universal measuring instrument Physics	531 835
or	
1 Mobile-CASSY	524 009
1 Combi-B-sensor S	524 0381
1 Extension cable, 15-pole	501 11
1 Stand base, V-shape, 20 cm	300 02
1 Stand rod, 47 cm	300 42
1 LD-multiclamp	301 01
3 Connecting leads, 100 cm	501 33
<i>add. required</i> : PC with Windows 98 or higher	

Safety notes

The electrical leads on the cadmium lamp and the resistors of the starting electrodes are open and easily accessible:

Do not touch any current-carrying (live!) parts.

Loose ferromagnetic objects can be strongly attracted by the electromagnet and can damage the quartz bulb of the cadmium lamp.

Check to make sure that the pole pieces are screwed tight before switching on the magnet current.

When the magnet current is switched on, do not handle ferromagnetic objects in the vicinity of the cadmium lamp.

Deposits of skin secretions can destroy the quartz bulb of the cadmium lamp when it becomes hot.

Never handle the quartz bulb of the cadmium lamp with your bare hands.

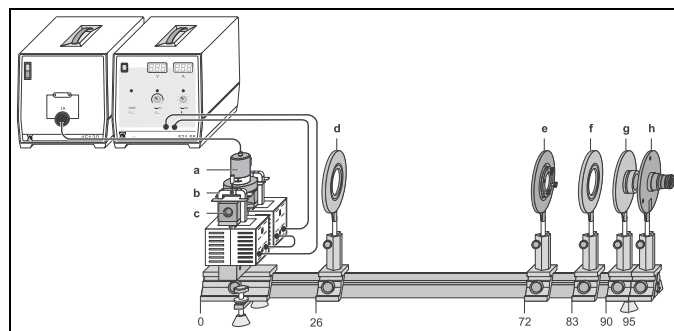
Set up

Fig. 3: Experimental setup for observing the *Zeeman* effect in transverse configuration. The position of the left edge of the optics riders is given in cm.

- a Cadmium lamp with holding plate
- b Clamps
- c Pole pieces
- d Positive lens, $f = 150$ mm (Condenser lens)
- e *Fabry-Perot* etalon
- f Positive lens, $f = 150$ mm (imaging lens)
- g Interference filter in holder
- h VideoCom (it is helpful to use an ocular for adjustment)

The complete experimental setup in transverse configuration is illustrated in Fig. 3.

Mechanical setup:

Screw the threaded rod into the base of the rider.

Put the U-yoke over the threaded rod on the base of the rider so that it is freely rotatable and put on the coils.

Mount the pole pieces and the holding plate of the cadmium lamp using the clamps so that a distance of approx. 10 mm is left between the pole pieces and that the opening of the holding plate points to the back. Do not yet fasten the screws of the clamps.

Cautiously insert the cadmium lamp between the pole pieces. See to it that the cadmium lamp is exactly in the middle of the pole pieces, that the point where the bulb is sealed off points to the back and that the supply leads are swivelled out of the ray path as far as possible. It must be possible to shift off and in the cadmium lamp with holding plate for measuring the magnetic field

Fix the pole pieces and the holding plate with the screws of the clamps.

Mount the B-sensor S with the help of the stand material, so that it is possible to hold the tangential probe vertically in the middle of the pole pieces.

Electrical connection:

Connect the coils of the electromagnet in series and then to the high current power supply.

Connect the B sensor S to the Universal measuring instrument Physics or the Mobile CASSY.

Connect the cadmium lamp to the universal choke.

Connect the VideoCom to the computer.

Adjusting the observing optics:

Mount the optical components according to Fig. 3.

After switching on the universal choke wait 5 min until the light emission is sufficiently strong.

If available, use first an ocular (e.g. 460 135 Ocular with scale) to get a sharp image of the ring system. Later on, the CCD of the VideoCom should be placed in the optical plane this image is located.

VideoCom:

It is possible to optimize the exposure time so that the peaks have an intensity of approximately 50 %.

To ensure that the CCD is in the focal plane of the imaging lens, move the imaging lens along the optical axis until the peaks of the observed curve are sharply imaged and show the maximum intensity. The center of the ring system must then be imaged on the CCD line. For this, you can either move VideoCom perpendicular to the optical axis or tilt the etalon slightly using the adjusting screws. You have found the center of the ring system when further adjustment does not cause any more peaks to emerge and the two central peaks (left and right intersections of the innermost rings) are the maximum distance apart.

Move the condenser lens until you obtain the most uniform possible illumination of the entire CCD line, so that the maxima in the middle are nearly at the same intensity (see Fig. 4).

Calibration of VideoCom

For angular calibration, the focal length of the imaging lens is specified (here $f = 150$ mm). To shift the zero point, determine the angles of the two central peaks using the function "Calculate Peak Center" and enter the negative mean value of the two peak centers manually. This sets the center of the ring system to 0° on the angular scale.

Carrying out the experiment**a) Measuring the magnetic field**

Hold the tangential probe vertically in the center of the pole pieces, while the cadmium lamp is not mounted.

Increase the current through the coils and measure the magnetic field in dependency of the current, see. Table 1.

b) Measuring the splitting of the cadmium line

Cautiously insert the cadmium lamp between the pole pieces without changing the distance of the pole pieces (see Mechanical setup). Start the software "VideoComInt" and adjust the observing optics.

The intensity of the peak should be around 50 % (adjust this if necessary as the luminance of the Cd lamp changes in the magnetic field).

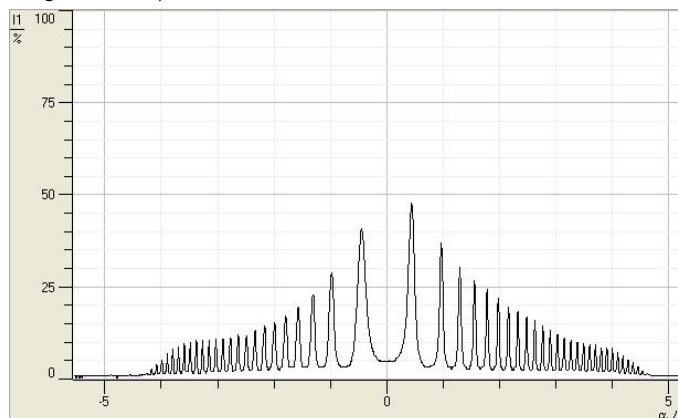


Fig.4: Intensity vs. angle without magnetic field

The table "Zeeman" is provided for evaluation; switch to this table by clicking on the corresponding tab. This table contains columns for the magnetic field B , the angle α_1 of the non-displaced line (π -component), the angle α_2 of the displaced line ($\sigma+$ or $\sigma-$) and the corresponding wavelength and energy shifts. These last two values are calculated automatically after entry of α_1 and α_2 using

$$\frac{\Delta\lambda}{\lambda} = \frac{\cos\beta_2}{\cos\beta_1} - 1 \quad \text{with} \quad \frac{\sin\alpha}{\sin\beta} = n = 1.46 \quad \text{for the etalon and}$$

$$\Delta E \approx \frac{-\Delta\lambda}{\lambda} \cdot E = -hc \cdot \frac{\Delta\lambda}{\lambda^2} \quad \text{with} \quad \lambda = 643.8 \text{ nm for the Cd}$$

line.

Start the evaluation by marking a peak (π) with "Calculate Peak Center". Transfer this value to the "Zeeman" table by dragging it from the status line to column α_1 of the table (drag and drop). To determine the origin in the diagram, drag the value to column α_2 as well and enter the value 0 in column B (non-displaced ring without magnetic field).

Next, examine the ring as it is split in the magnetic field ($\sigma+$ or $\sigma-$). Once again, determine the displaced angle α_2 using "Calculate Peak Center" and drag it to column α_2 in the "Zeeman" table. The non-displaced angle α_1 remains unchanged (you can e.g. import it from a previous measurement using drag and drop), and the magnetic flux density is entered. Using these values, the software calculates the percentage shift in wavelength and the energy split. Additionally, the corresponding point is plotted in the diagram. (To confirm the effect for other rings at equal flux densities or for repetition with different flux densities, it is necessary to examine peaks with identical components ($\sigma+$ or $\sigma-$) each time.)

To get the gradients of the line splitting fit straight lines through the measured points.

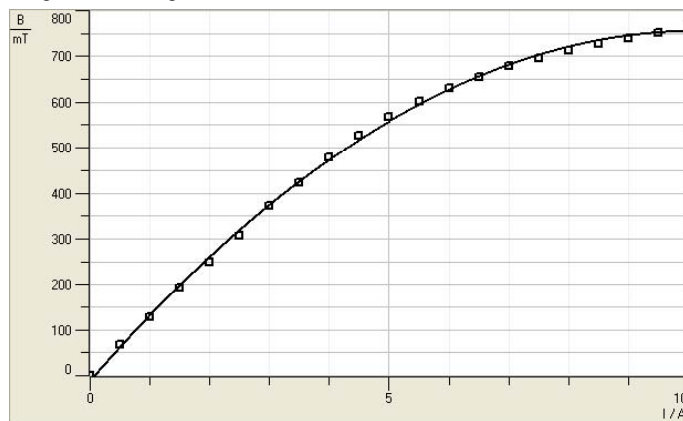
Measuring example

a) Measuring the magnetic field

Tab. 1: Magnetic field *B* vs. current *I*

$\frac{I}{A}$	$\frac{B}{mT}$	$\frac{I}{A}$	$\frac{B}{mT}$
0.0	0		
0.5	68	5.5	601
1.0	127	6.0	631
1.5	194	6.5	655
2.0	249	7.0	678
2.5	307	7.5	695
3.0	373	8.0	713
3.5	423	8.5	728
4.0	479	9.0	739
4.5	528	9.5	752
5.0	569	10.0	765

Diagramm 1: Magnetic field *B* vs. current *I*



Evaluation

The gradients *A* for σ_+ or σ_- components of the curves are:

$$A = 56.7 \frac{\mu\text{eV}}{\text{T}} \text{ and } A = -58.8 \frac{\mu\text{eV}}{\text{T}}$$

The mean value gives *Bohr's* magneton:

$$\mu_B = 57.8 \frac{\mu\text{eV}}{\text{T}} = 9.26 \cdot 10^{-24} \frac{\text{Js}}{\text{T}}$$

With $h = 6.626 \cdot 10^{-34} \text{ Js}$:

$$\frac{e}{m_e} = \frac{4\pi \mu_B}{h} = 1.76 \cdot 10^{11} \frac{\text{C}}{\text{kg}}$$

Literature values:

$$\mu_B = 57.9 \frac{\mu\text{eV}}{\text{T}}$$

$$\frac{e}{m_e} = 1.759 \cdot 10^{11} \frac{\text{C}}{\text{kg}}$$

b) Measuring the splitting of the cadmium line

Fig. 5: Screenshot "Zeeman" table with diagramm

