Atomic and nuclear physics

X-ray physics
Physics of the atomic shell

Moseley's law and determination of the Rydberg constant

Objects of the experiment

- Measuring the K-absorption edges in the transmission spectra of Zr, Mo, Ag and In.
- Verifying Moseley's law.
- Determining the Rydberg constant.

Principles

The absorption of x-ray quanta during the passage of x-rays through matter is essentially due to ionization of atoms, which release an electron from an inner shell, e.g. the K-shell. This can only occur when the quantum energy

$$E = \frac{h \cdot c}{\lambda} \tag{I}$$

h: Planck's constant,
c: velocity of light

is greater than the binding energy E_K of the shell. The transmission

$$T = \frac{R}{R_0} \tag{II}$$

R: intensity rate behind attenuator
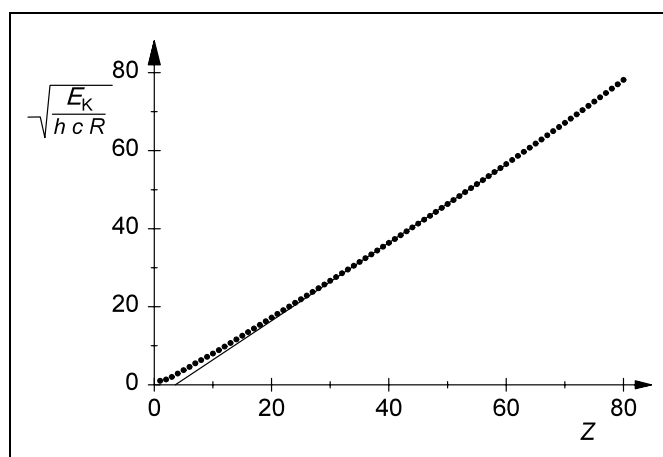R_0: intensity in front of attenuator

Fig. 1   Binding energy of the K-shell in the form

$\sqrt{\frac{E_K}{h \cdot c \cdot R}}$ as a function of the atomic number Z

(see also equation (V))
Circles: Literature values from [1]
Line: best-fit straight line for $Z = 30$–60

of the material thus increases abruptly as a function of the wavelength at

$$\lambda_K = \frac{h \cdot c}{E_K} \tag{III}$$

This abrupt change is known as an absorption edge, here the K-absorption edge.

In 1913, the English physicist Henry Moseley measured the K-absorption edges for various elements and formulated the law that bears his name:

$$\sqrt{\frac{1}{\lambda_K}} = \sqrt{R} \cdot (Z - \sigma_K) \tag{IV}$$

R: Rydberg constant
Z: atomic number of absorbing elements
σ_K: screening coefficient of K-shell

A comparison with (III) gives us a value for the binding energy of the K-edge of

$$E_K = h \cdot c \cdot R \cdot (Z - \sigma_K)^2 \tag{V}$$

We can bring this equation into agreement with the predictions of Bohr's model of the atom when we consider the following: The nuclear charge $Z \cdot e$ of an atom is partially screened from the electron ejected from the K-shell through absorption of the x-ray quantum by the remaining electrons of the atomic shell. Therefore, on average, only the charge $(Z - \sigma_K) \cdot e$ acts on the electron during ionization.

This experiment verifies Moseley's law by measuring the K-absorption edges for the atomic numbers Z between 40 and 50. In this range, the screening coefficient σ_K is largely independent of Z (see Fig. 1). Thus, (IV) is equivalent to a general, straight-line equation in the form

$$y = a \cdot x + b \tag{VI}$$

with the atomic number Z as the x-variable. From the parameters a and b of the straight line, we can calculate the Rydberg constant R and the screening coefficient σ_K:

$$R = a^2, \; \sigma_K = -\frac{b}{a} \tag{VII}.$$

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Apparatus

1 X-ray apparatus 554 811

1 End-window counter
   for α, β, γ and x-ray radiation 559 01

1 Set of absorber foils 554 832
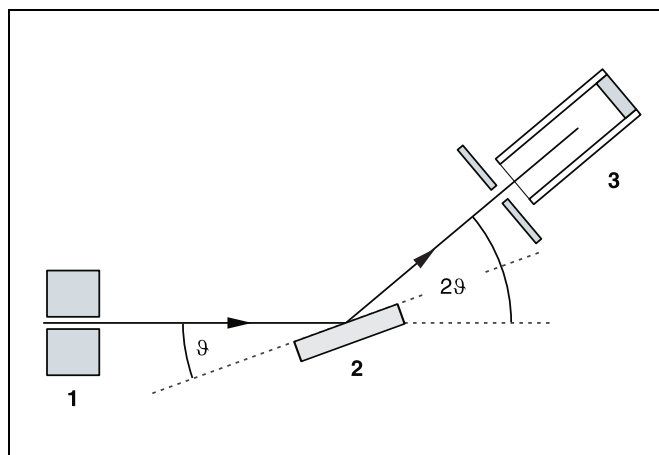
additionally required:

1 PC with Windows 9x/NT

Fig. 2   Schematic diagram of diffraction of x-rays at a monocrystal and 2ϑ coupling between counter-tube angle and scattering angle (glancing angle)
       1 collimator, **2** monocrystal, **3** counter tube

A goniometer with NaCl crystal and a Geiger-Müller counter tube in the Bragg configuration are used to measure the transmission T as a function of the wavelength. The crystal and counter tube are pivoted with respect to the incident x-ray beam in 2ϑ coupling, i. e. the counter tube is turned at an angle twice as large as the crystal (see Fig. 2).

In accordance with Bragg's law of reflection, the scattering angle ϑ in the first order of diffraction corresponds to the wavelength

$$\lambda = 2 \cdot d \cdot \sin \vartheta \tag{VIII}.$$
$d = 282.01$ pm: lattice plane spacing of NaCl

Safety notes

The x-ray apparatus fulfills all regulations governing an x-ray apparatus and fully protected device for instructional use and is type approved for school use in Germany (NW 807/97 Rö).

The built-in protection and screening measures reduce the local dose rate outside of the x-ray apparatus to less than 1 μSv/h, a value which is on the order of magnitude of the natural background radiation.

■   Before putting the x-ray apparatus into operation inspect it for damage and to make sure that the high voltage is shut off when the sliding doors are opened (see Instruction Sheet for x-ray apparatus).

■   Keep the x-ray apparatus secure from access by unauthorized persons.

Do not allow the anode of the x-ray tube Mo to overheat.

■   When switching on the x-ray apparatus, check to make sure that the ventilator in the tube chamber is turning.

The goniometer is positioned solely by electric stepper motors.

■   Do not block the target arm and sensor arm of the goniometer and do not use force to move them.

Setup

Setup in Bragg configuration:

Set up the experiment as shown in Fig. 3. To set up the experiment, proceed as follows (see also the Instruction Sheet for the x-ray apparatus):

–   Mount the collimator in the collimator mount **(a)** (note the guide groove).

–   Attach the goniometer to guide rods **(d)** so that the distance s_1 between the slit diaphragm of the collimator and the target arm is approx. 5 cm. Connect ribbon cable **(c)** for controlling the goniometer.

–   Remove the protective cap of the end-window counter, place the end-window counter in sensor seat **(e)** and connect the counter tube cable to the socket marked GM TUBE.

–   By moving the sensor holder **(b)**, set the distance s_2 between the target arm and the slit diaphragm of the sensor seat to approx. 5 cm.

–   Mount the target holder with target stage.

–   Loosen knurled screw **(g)**, place the NaCl crystal flat on the target stage **(f)**, carefully raise the target stage with crystal all the way to the stop and carefully tighten the knurled screw (prevent skewing of the crystal by applying a slight pressure).

–   If necessary, adjust the mechanical zero position of the goniometer (see Instruction Sheet for x-ray apparatus).

Notes:

NaCl crystals are hygroscopic and extremely fragile.

Store the crystals in a dry place; avoid mechanical stresses on the crystal; handle the crystal by the short faces only.

If the counting rate is too low, you can reduce the distance s_2 between the target and the sensor somewhat. However, the distance should not be too small, as otherwise the angular resolution of the goniometer is no longer sufficient to separate the characteristic K_α and K_β lines.
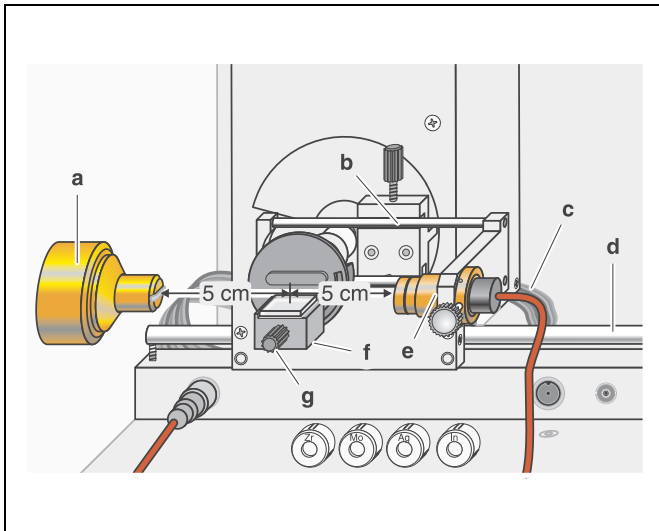
Fig. 3   Experiment setup for measuring the K-absorption edges.

Preparing the PC-based measurement:

– Connect the RS–232 output and the serial interface on your PC (usually COM1 or COM2) using the 9-pin V.24 cable (supplied with x-ray apparatus).
– If necessary, install the software "X-ray Apparatus" under Windows 9x/NT (see Instruction Sheet for x-ray apparatus) and select the desired language.

Carrying out the experiment

– Start the software "X-ray Apparatus", check to make sure that the apparatus is connected correctly, and clear any existing measurement data using the button [image] or the F4 key.
– Set the tube high voltage $U = 35.0$ kV, the emission current $I = 1.00$ mA and the angular step width $\Delta\beta = 0.1°$.
– Press the COUPLED key to activate 2ϑ coupling of target and sensor and set the lower limit of the target angle to 3.7° and the upper limit to 7.5°.
– Set the measuring time per angular step to $\Delta t = 5$ s.
– Start measurement and data transfer to the PC by pressing the SCAN key.
– When the scan is finished, mount the zirconium foil on sensor seat **(e)** of the goniometer and start a new measurement by pressing the SCAN key.
– Replace the Zr foil with the Mo, Ag and In foils one after another and conduct further measurements.

– When you have finished measuring, save the measurement series under an appropriate name by pressing the button [image] or the F2 key.

– To display the measurement data as a function of the wavelength λ, open the "Settings" dialog with the button [image] or F5, and in the tab "Crystal", click on the button "Enter NaCl".

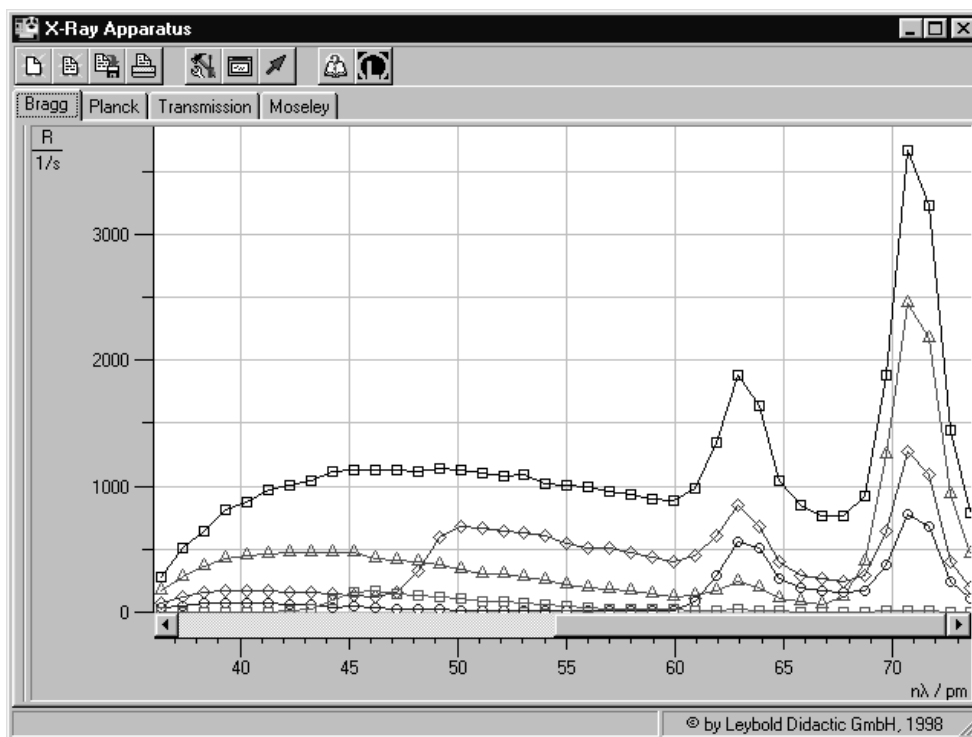## Measuring example

(see Fig. 4)

Fig. 4   Diffraction spectra of x-rays with the characteristic lines of the molybdenum anode in the wavelength range between 36 and 74 pm ($U = 35$ kV)
Squares: Not attenuated
Triangles: Attenuation in Zr foil
Circles: Attenuation in Mo foil
Hashes: Attenuation in Ag foil
Squares: Attenuation in In foil

Evaluation

– Click on the "Transmission" tab in the software "X-ray Apparatus" to generate the transmission spectra (see Fig. 5) from the diffraction spectra (see Fig. 4).

– In the diagram, click the right mouse button to access the evaluation functions of the software "X-ray Apparatus" and select the command "Draw K-Edges".

– Mark the range of each K-edge in the transmission spectra using the left mouse button.

– Click on the tab "Moseley" and enter the atomic numbers of the respective foils (Zr: 40, Mo: 42, Ag: 47 and In: 49) in the column Z (see Fig. 6).

– Place the mouse pointer over the diagram, click the right mouse button and select the evaluation command "Best-fit Straight Line"; then hold down the left mouse button to mark the range in the diagram to which you want to fit the line. Read off the result for the Rydberg constant R and the screening coefficient σ_K in the bottom left corner of the window.

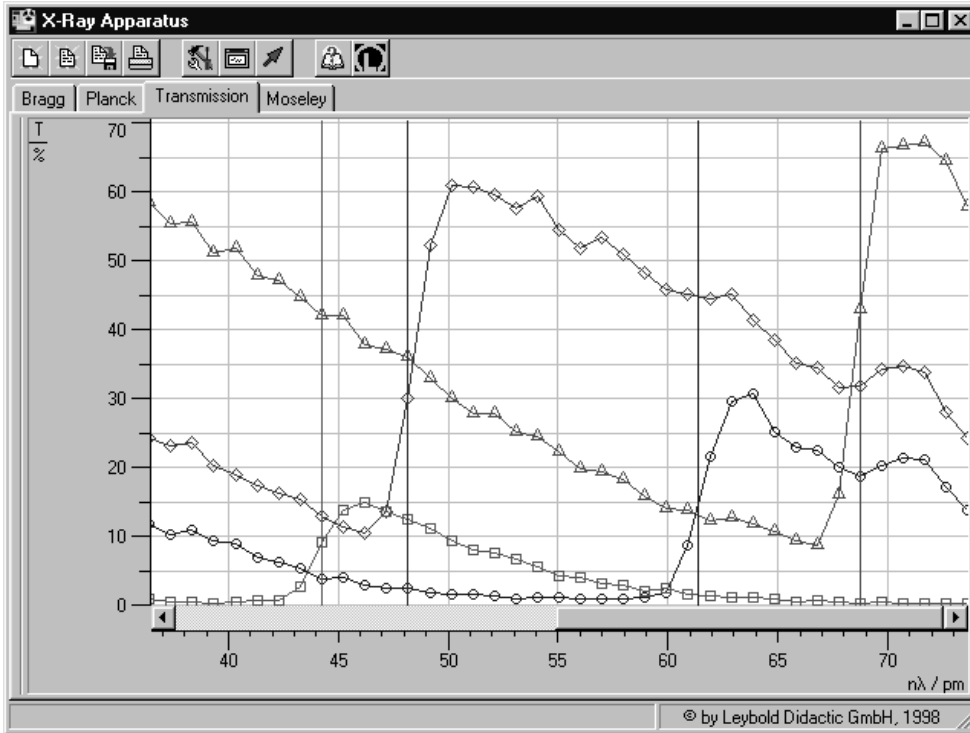Fig. 6 shows how the result should look.

Fig. 5   Transmission spectra in the wavelength range between 36 and 74 pm
Triangles:
Attenuation in Zr foil
Circles:
Attenuation in Mo foil
Hashes:
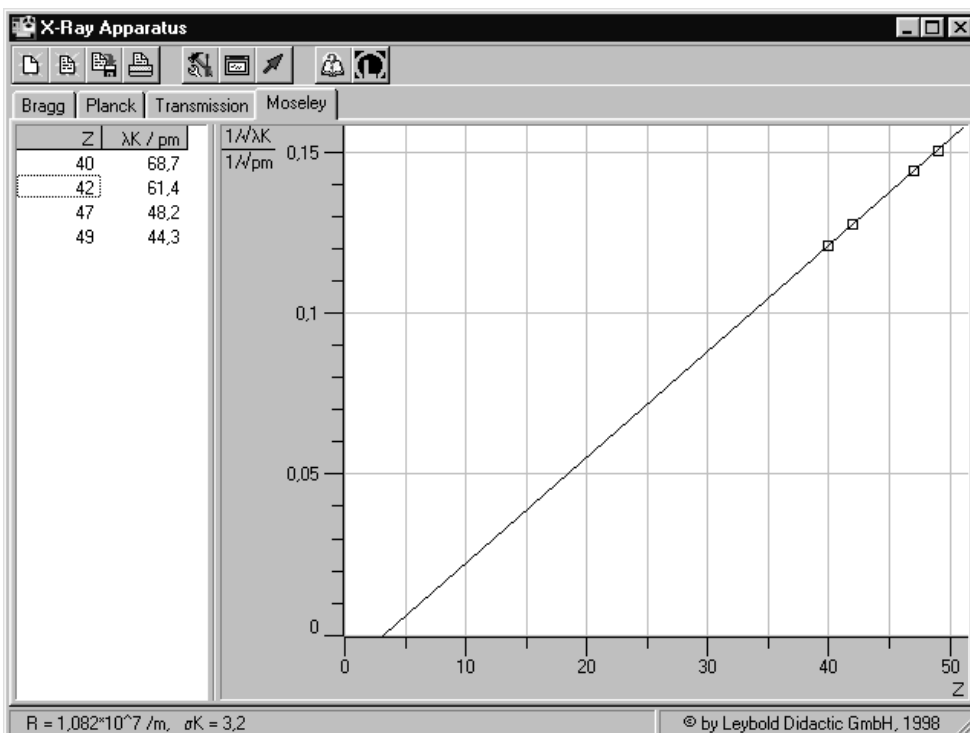Attenuation in Ag foil
Squares:
Attenuation in In foil

Fig. 6   Evaluation of measurement data for verification of Moseley's law

Results

The result of the graphical evaluation is:

$R = 1.082 \cdot 10^7$ m^{-1}, $\sigma_K = 3.2$

Literature values [1]:

$R = 1.097373 \cdot 10^7$ m^{-1}

$\sigma_K = 3.6$ (for moderately heavy nuclei)

Literature

[1] C. M. Lederer and V. S. Shirley, Table of Isotopes, 7th Edition, 1978, John Wiley & Sons, Inc., New York, USA.